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THERMOELASTIC DEFORMATION OF A COOLED METAL PLATE UNDER THE INFLUENCE OF A PULSE-PERIODIC RADIATION FLUX

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A solution to the problem of determining the fields of stress and deformation in a plate under the influence of radiation flux with a Gaussian distribution is obtained.

A common element in optical systems is a metal plate, the surface of which has a high coefficient of reflection as a result of processing. Under the influence of a sufficiently high radiation flux density on the plate, the planar reflecting surface buckles due to nonuniform heating. This leads to a change in the structure of the beam; in particular, defocusing occurs as a result of reflection from such a surface [1]. In addition, thermal stress develops in the plate. During intense heating the magnitude of this stress can exceed the tensile strength of the plate material, thereby inducing an irreversible structural change.

In [2] a calculation of the thermal stress in a cooled plate under the influence of a pulse-periodic radiation flux was performed within a one-dimensional approximation where the stress tensor components and temperature change in the direction normal to the surface of the plate. In [3] a relation for the temperature fields in a plate was obtained within the one-dimensional approximation, and an estimation of the normal deformation and stress was performed. In [4] the two-dimensional problem of stress location in a free round plate under a radially Gaussian distributed radiation flux density was determined. It was shown that the structure of the spatial distribution of the stress within the one- and two-dimensional cases differs significantly. In particular, it was found that in the center zone of irradiation, the tangential and axial components of the stress are compressing, but out of the zone of irradiation they are stretching.

In the present work, in contrast to [4], the primary emphasis is the deformation of the plate surface induced by the thermal effect of a pulse-periodic radiation flux with a radial Gaussian distribution. We will assume that the rear surface of the plate is fixed to a rigid base, and the heat transfer from it proceeds according to Newton's law.

We will find the temperature field of a plate of constant thickness d and infinite in the radial direction. One of the surfaces of the plate ($z = 0$) is heated as a result of the influence of the pulse-periodic source (radiation directed along the normal to the surface), and the other ($z = d$) is cooled by means of a cooling agent with a coefficient of heat transfer h. We will assume that the intensity of the surface thermal source can be represented in the form

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$$Q(r,\ t)=I_0\exp(-ar^2)f(t).$$

The function f(t) gives the time dependence of the intensity of the thermal source

$$f(t)=\begin{cases}1,\ (N-1)(\tau+\Delta)<t<(N-1)(\tau+\Delta)+\tau,\\ 0,\ (N-1)(\tau+\Delta)+\tau<t<N(\tau+\Delta),\end{cases}\tag{1}$$

where N is the number of pulses in a series.

We find the temperature field on the basis of the solution of the heat-transfer equation with appropriate boundary conditions

$$\begin{gathered}\frac{\partial T}{\partial t}=k\left(\frac{\partial^2T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2T}{\partial z^2}\right),\\ -\lambda\frac{\partial T}{\partial z}\bigg|_{z=0}=I_0\exp(-ar^2)f(t),\\ -\lambda\frac{\partial T}{\partial z}\bigg|_{z=d}=hT|_{z=d},\ T|_{t=0}=0.\end{gathered}\tag{2}$$

Applying Hankel and Laplace transforms to the heat-transfer equation and the boundary conditions leads to

$$\begin{gathered}s\overline{T}+kp^2\overline{T}-k\frac{\partial^2\overline{T}}{\partial z^2}=0,\\ -\lambda\frac{\partial\overline{T}}{\partial z}\bigg|_{z=0}=I_0\frac{1}{2a}\exp\left(-\frac{p^2}{4a}\right)\{f(t)\}_L,\quad -\lambda\frac{\partial\overline{T}}{\partial z}\bigg|_{z=d}=h\overline{T}|_{z=d},\end{gathered}\tag{3}$$

where $\overline{T}=\int\limits_0^\infty e^{-st}\int\limits_0^\infty rJ_0(pr)T(r,\ z,\ t)\,drdt$; p and s are parameterss of the Hankel and Laplace transforms; $\{f(t)\}_L$ is the Laplace transform of relation (1). As a result of Eq. (3), we obtain

$$\begin{gathered}\overline{T}=\frac{I_0d}{2a\lambda}\exp\left(-\frac{p^2}{4a}\right)\{f(t)\}_L\frac{\mathrm{Bi}}{\mu(\mu\operatorname{sh}\mu+\mathrm{Bi}\operatorname{ch}\mu)}\left\{\left(\frac{\mu}{\mathrm{Bi}}\times\right.\right.\\ \left.\left.\times\operatorname{ch}\mu+\operatorname{sh}\mu\right)\operatorname{ch}\mu\frac{z}{d}-\left(\frac{\mu}{\mathrm{Bi}}\operatorname{sh}\mu+\operatorname{ch}\mu\right)\operatorname{sh}\mu\frac{z}{d}\right\},\ \mu=\sqrt{\frac{s}{k}+p^2d}.\end{gathered}\tag{4}$$

To transform from representation (4) to the original expression, we use the theorem of decomposition and expansion [5]:

$$T(r,\ z,\ t)=\frac{I_0k}{a\lambda d}\int\limits_0^\infty\exp\left(-\frac{p^2}{4a}\right)pJ_0(pr)\sum_{n=1}^\infty\gamma_n\cos\psi_n\frac{z}{d}\left\{\int\limits_0^t\exp[-\beta_n(t-\theta)]f(\theta)\,d\theta\right\}dp,\tag{5}$$

where ψ_n is a root of the characteristic equation $\cot\psi_n=\psi_n/\mathrm{Bi}$;

$$\beta_n=\frac{k}{d^2}(\psi_n^2+p^2d^2),\ \gamma_n=\frac{\psi_n}{\psi_n+\cos\psi_n\sin\psi_n}.$$

Using expression (1) for f(t), we compute the inner integral in (5):

$$\begin{gathered}\int\limits_0^t\exp[-\beta_n(t-\theta)]f(\theta)\,d\theta=\exp(-\beta_nt)\left[\int\limits_0^\tau\exp(\beta_n\theta)\,d\theta+\right.\\ \left.+\int\limits_{\tau+\Delta}^{2\tau+\Delta}\exp(\beta_n\theta)\,d\theta+\int\limits_{2(\tau+\Delta)}^{3\tau+2\Delta}\exp(\beta_n\theta)\,d\theta+\ldots\right]=\frac{1}{\beta_n}\exp(-\beta_nt)F_n(t),\end{gathered}\tag{6}$$

where

$$F_n(t) = [\exp \beta_n \tau - 1] \frac{\exp \beta_n N(\tau + \Delta) - 1}{\exp \beta_n (\tau + \Delta) - 1} + \exp \beta_n t - \exp \beta_n [N\tau + (N-1)\Delta].$$

Taking into account (6), expression (5) takes the form

$$T(r, z, t) = \frac{I_0 d}{a\lambda} \int_0^\infty \exp\left(-\frac{p^2}{4a}\right) p J_0(pr) \left[\sum_{n=1}^\infty \frac{\gamma_n F_n(t)}{\psi_n^2 + p^2 d^2} \cos \psi_n \frac{z}{d} \exp(-\beta_n t)\right] dp. \tag{7}$$

We will perform a calculation of the components of transfer u, w corresponding to the radial and axial directions, which satisfy the following equations [6]:

$$\Delta u - \frac{u}{r^2} + \frac{1}{1-2\nu} \frac{\partial e}{\partial r} = \frac{2(1+\nu)}{1-2\nu} \alpha \frac{\partial T}{\partial r}, \tag{8}$$

$$\Delta w + \frac{1}{1-2\nu} \frac{\partial e}{\partial z} = \frac{2(1+\nu)}{1-2\nu} \alpha \frac{\partial T}{\partial z}, \tag{9}$$

where $e = \partial u/\partial r + u/r + \partial w/\partial z$ is the volume expansion. We represent u, w in the form

$$u(r, z, t) = \frac{I_0 d}{a\lambda} \int_0^\infty \exp\left(-\frac{p^2}{4a}\right) J_1(pr) \sum_{n=1}^\infty \frac{\gamma_n \varphi_n(z)}{\psi_n^2 + p^2 d^2} F_n(t) \exp(-\beta_n t)\, dp, \tag{10}$$

$$w(r, z, t) = \frac{I_0 d}{a\lambda} \int_0^\infty \exp\left(-\frac{p^2}{4a}\right) J_0(pr) \sum_{n=1}^\infty \frac{\gamma_n \Phi_n(z)}{\psi_n^2 + p^2 d^2} F_n(t) \exp(-\beta_n t)\, dp. \tag{11}$$

The unknown functions $\varphi_n(z)$ and $\Phi_n(z)$ satisfy the system (12), (13), which is obtained as a result of substitution of relations (10) and (11) into (8) and (9):

$$-\frac{d^2 \varphi_n}{dz^2} + 3p \frac{d\Phi_n}{dz} + 4p^2 \varphi_n = 8\alpha p^2 \cos \psi_n \frac{z}{d}, \tag{12}$$

$$4 \frac{d^2 \Phi_n}{dz^2} + 3p \frac{d\varphi_n}{dz} - p^2 \Phi_n = -8\alpha p \frac{\psi_n}{d} \sin \psi_n \frac{z}{d}. \tag{13}$$

Here it is assumed that $1/(1-2\nu) = 3$, $(1+\nu)/(1-2\nu) = 4$ since for many metals of practical interest, $\nu = 0.33$. We differentiate (12) with respect to the variable z and combine with Eq. (13):

$$\frac{d^2}{dz^2}\left(\frac{d\varphi_n}{dz} + p\Phi_n\right) - p^2\left(\frac{d\varphi_n}{dz} + p\Phi_n\right) = 0.$$

The solution of this equation is written in the form

$$\frac{d\varphi_n}{dz} + p\Phi_n = A_n e^{-pz} + B_n e^{pz}, \tag{14}$$

where A_n and B_n are arbitrary constants to be determined from the boundary conditions. Substituting (14) into (12), we obtain

$$\frac{d\Phi_n}{dz} + p\varphi_n = 2\alpha p \cos \psi_n \frac{z}{d} + \frac{1}{4}(-A_n e^{-pz} + B_n e^{pz}). \tag{15}$$

We find the desired functions φ_n and Φ_n from the solution of the system of differential equations (14) and (15)

$$\Phi_n = 2\alpha p \frac{\psi_n d}{\psi_n^2 + p^2 d^2} \sin \psi_n \frac{z}{d} + e^{-pz}\left[\frac{3}{8} A_n z + \frac{5}{16} \frac{A_n}{p} + \frac{1}{2} C_n\right] + e^{pz}\left(-\frac{3}{8} B_n z + \frac{5}{16} \frac{B_n}{p} + \frac{1}{2} D_n\right), \tag{16}$$

$$\varphi_n = 2\alpha \frac{p^2 d^2}{\psi_n^2 + p^2 d^2} \cos \psi_n \frac{z}{d} + e^{-pz}\left(\frac{3}{8} A_n z - \frac{5}{16} \frac{A_n}{p} + \frac{1}{2} C_n\right) + e^{pz}\left(\frac{3}{8} B_n z + \frac{5}{16} \frac{B_n}{p} - \frac{1}{2} D_n\right).$$

Here C_n and D_n are arbitrary constants to be determined. We find the coefficients A_n, B_n, C_n and D_n from the boundary conditions, the form of which depends on the method of fixing the plate to the base. We consider the case where the plate is fixed securely to a rigid base. Then we have the next condition:

for z = 0

$$\sigma_{rz} = G\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right) = 0, \ \sigma_{zz} = 2G\left(\frac{\partial w}{\partial z} + e - 4\alpha T\right) = 0;$$

for z = d

$$u = w = 0.$$

Here σ_{rz}, σ_{zz} are components of the stress tensor. Taking into account (10) and (11), the boundary conditions take the form

for z = 0

$$\frac{d\varphi_n}{dz} - p\Phi_n = 0, \quad 2\frac{d\Phi_n}{dz} + p\varphi_n = 4\alpha p;$$

for z = d

$$\varphi_n = \Phi_n = 0. \tag{17}$$

From relations (17) and (16), we obtain a system of algebraic equations relative to A_n, B_n, C_n, and D_n:

$$\begin{gathered}
\frac{3}{8}(A_n + B_n) - p(C_n + D_n) = 0,\\
\frac{3}{16}(B_n - A_n) + \frac{1}{2}p(D_n - C_n) = 2\alpha p\frac{x^2}{\psi_n^2 + x^2},\\
A_n\left(\frac{3}{4}x + \frac{5}{8}\right)e^{-x} + B_n\left(\frac{5}{8} - \frac{3}{4}x\right)e^{x} + C_n p e^{-x} + D_n p e^{x} = -4\alpha p\frac{x\psi_n \sin\psi_n}{\psi_n^2 + x^2},\\
A_n\left(\frac{3}{4}x - \frac{5}{8}\right)e^{-x} + B_n\left(\frac{5}{8} + \frac{3}{4}x\right)e^{x} + C_n p e^{-x} - D_n p e^{x} = -4\alpha p\frac{x^2\cos\psi_n}{\psi_n^2 + x^2},
\end{gathered} \tag{18}$$

where x = pd. Substituting into relation (16) the coefficients A_n, B_n, C_n, and D_n, found from the solution of (18), we obtain the value of φ_n and Φ_n. The transfer components u, w, in accordance with (10) and (11), depend on φ_n and Φ_n, but the stress tensor components are expressed in terms of u, w and their first derivatives through the well-known correlation of Hooke's law [6]. We reduce the expression for w at z = 0 and also σ_{rz}, σ_{zz} for z = d:

$$w(r,\ 0,\ t) = \frac{I_0 d}{2a\lambda}\int_0^\infty \exp\left(-\frac{x^2}{4ad^2}\right)J_0\left(x\frac{r}{d}\right)\frac{1}{x}\sum_{n=1}^\infty \gamma_n\frac{A_n + B_n}{\psi_n^2 + x^2}F_n(t)\exp(-\beta_n t)\,dx,$$

$$\sigma_{zz}(r,\ d,\ t) = \frac{GI_0}{a\lambda}\int_0^\infty \exp\left(-\frac{x^2}{4ad^2}\right)J_0\left(x\frac{r}{d}\right)\sum_{n=1}^\infty \gamma_n\frac{B_n e^{x} - A_n e^{-x}}{\psi_n^2 + x^2}F_n(t)\exp(-\beta_n t)\,dx,$$

$$\sigma_{rz}(r,\ d,\ t) = \frac{GI_0}{a\lambda}\int_0^\infty \exp\left(-\frac{x^2}{4ad^2}\right)J_1\left(x\frac{r}{d}\right)\sum_{n=1}^\infty \gamma_n\frac{A_n e^{-x} + B_n e^{x}}{\psi_n^2 + x^2}F_n(t)\exp(-\beta_n t)\,dx,$$

where

$$A_n = \frac{4\alpha p x}{M(\psi_n^2 + x^2)}\left\{[(\mathrm{Bi} + x)\cos\psi_n - xe^{-x}]\frac{3}{2}xe^{x} + [(\mathrm{Bi} - x)\cos\psi_n + xe^{x}]\left(\frac{5}{4}e^{x} + \frac{3}{4}e^{-x}\right)\right\},$$

$$B_n = \frac{4\alpha p x}{M(\psi_n^2 + x^2)}\left\{[(\mathrm{Bi} + x)\cos\psi_n - xe^{-x}]\left(\frac{5}{4}e^{-x} + \frac{3}{4}e^{x}\right) - \frac{3}{2}xe^{-x}[(\mathrm{Bi} - x)\cos\psi_n + xe^{x}]\right\},$$

$$M = -\left[\frac{9}{4}x^2 + \left(\frac{5}{4}e^{-x} + \frac{3}{4}e^{x}\right)\left(\frac{5}{4}e^{x} + \frac{3}{4}e^{-x}\right)\right]. \tag{19}$$

As is well known, the time dependence of the temperature on the reflecting surface of the plate under the influence of a pulse-periodic flux is oscillatory [2, 3, 7], while the period of the oscillation is equal to $\Delta + \tau$. For this reason, the values of w, σ_{zz}, and σ_{rz} also oscillate. Figure 1 shows the dependence on Bi of the displacement of the face of the surface from the middle of the plate w(0, 0, t), obtained from (19) for the following values: $\Delta = 0.5 \cdot 10^{-2}$ sec, $\tau = 10^{-4}$ sec, $a = 8$ cm^{-2}, d = 0.1 cm, k = 1 cm^2/sec. From analysis of expression (7) it follows that in order to heat the plate surface up to a maximum temperature T_m, a certain number of pulses of radiation is necessary, satisfying the relation $\frac{k}{d^2}\psi_1^2[N_m\tau + (N_m - 1)\Delta] \geqslant 5$. Hence, for plate thickness d = 0.1 cm and Bi = 0.05 ($\psi_1 = 0.22$) the number of pulses needed is $N_m > 200$; if Bi = 0.15 ($\psi_1 = 0.37$), then $N_m > 70$. In Fig. 2, which is obtained from (19), we show the dependence of the displacement w(0, 0, t) on the parameter a, which characterizes the radial distribution of the intensity of the thermal source (the values of Δ, τ, and d as given above). From Fig. 3 it follows that the tensor components of the thermal stress σ_{rz} and σ_{zz} have different signs for z = d. This arises from the fact that the stress σ_{zz} is stretching, but σ_{rz} is compressing. The tensile strength of copper for stretching (stretching strength) is several times lower than for compressing (compression strength) and measures ~20 MPa. As is seen in Fig. 3, for E > 2 J the stress σ_{zz} surpasses the tensile strength, leading to a breakdown of the plate material or to its separation from the base.

As a result of deflection from the front surface of the plate, initially the parallel beam spreads. For the sake of computing the optical force on the reflecting surface for paraxial rays in expression (19) for w(r, 0, t), we expand the Bessel function in a series, restricted to the quadratic term:

$$J_0\left(x\frac{r}{d}\right) = 1 - \frac{1}{4}\left(x\frac{r}{d}\right)^2 + \ldots \tag{20}$$

Then the reflecting surface of the plate represents a paraboloid of rotation $r^2 = 2lz$ (l is the parameter of the paraboloid); the focal distance is $F = l/2$. From (19), taking into account (20), we obtain the following relation for the optical force:

$$|F|^{-1} = -\frac{E}{2\pi\lambda d\tau}\int_0^\infty \exp\left(-\frac{x^2}{4ad^2}\right) x \sum_{n=1}^{\infty} \gamma_n \frac{A_n + B_n}{\psi_n^2 + x^2} F_n(t) \exp(-\beta_n t)\, dx.$$

A curve showing the dependence of $|F|^{-1}$ on Bi is given in Fig. 1.

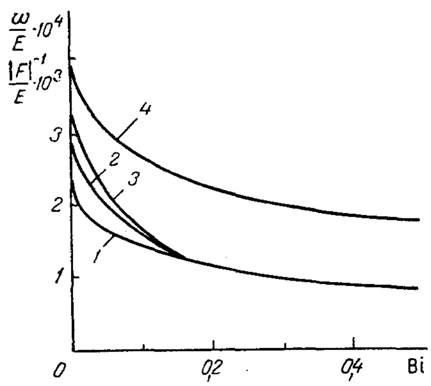

Fig. 1

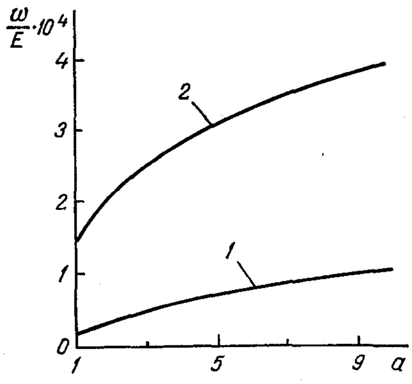

Fig. 2

Fig. 1. Dependence of normal travel w(0, 0, t) (cm) (curves 1-3) and optical force $|F|^{-1}$ (cm^{-1}) (4) on Bi under the effect of N = 25 (1), 50 (2), 100 (3, 4) pulses.

Fig. 2. Dependence of normal travel w(0, 0, t) (cm) on the parameter a (cm^{-2}) at Bi = 0.3 (1) and Bi = 0 (2) (N = 100).

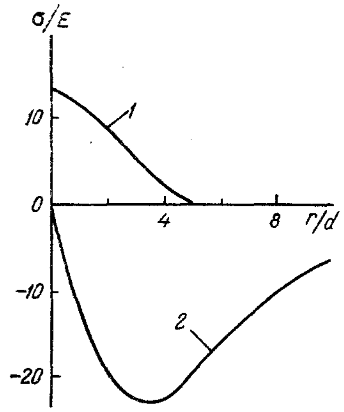

Fig. 3. Dependence of the components of thermoelastic stresses σ_{zz} (1) and σ_{rz} (2) (MPa) on the radial coordinate r (cm) at Bi = 0.05 (N = 100).

The previously derived relation (19) can also be utilized in the case where the beam has a central circular shading of radius b [3]. In this case it is necessary to make the substitution

$$\exp\left(-\frac{x^2}{4ad^2}\right) \to \exp\left(-\frac{x^2}{4ad^2}\right) - \int\limits_0^{ab^2} \exp(-y) J_0\left(\frac{x}{d}\sqrt{\frac{y}{a}}\right) dy$$

in the integral expressions. This substitution results immediately from the Hankel transform of the boundary conditions (2) on the reflecting surface.

NOTATION

t, time; r, z, radial and axial coordinates; I_0, radiation flux density; k, λ, thermal diffusivity and thermal conductivity; h, heat-transfer coefficient; Bi = hd/λ, Biot number; Δ, time between pulses; τ, pulse duration; $E = \pi I_0 \tau / a$, pulse energy absorbed by the plate, J; α, temperature coefficient of linear expansion; ν, Poisson coefficient; G, shear modulus; $J_m(x)$, Bessel function of the first kind of order m = 0, 1.

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